Lab Assignment No. 1: Answer Key

1) Given the matrix K, identify elements k_{31}, and k_{23}.

$$K = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix} \qquad K[3,1] = 0 \qquad K[2,3] = 0$$

2) What would a vector (vk) created from the diagonal of matrix K look like?

$$vk = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

3) What is the product of vector vk post multiplied by the corresponding transposed 1-vector?

$$\underline{1} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \qquad \underline{1}' = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$$

$$vk \times \underline{1}' = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \times \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

4) Is the result in Question 3 an identity matrix? Explain?

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad (vk \times \underline{1}') - I = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 0 & -1 \end{bmatrix}$$

No, it is not an identity matrix. If it was an identity matrix the difference between the product in question 3 and the expected identity matrix should produce a null matrix, which it did not.

5) What is the product of vector vk pre multiplied by the corresponding transposed 1-vector?

$$\underline{1}' \times vk = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1\times 1 + 1\times 1 + 1\times 0 \end{bmatrix} = \begin{bmatrix} 2 \end{bmatrix}$$

6) What is the trace of matrix K?

$$K = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix} \qquad tr(K) = 1 + 1 + 0 = 2$$

7) What is the determinant of matrix K?

In order to get the determinant we will expand by the first row and compute the determinant based on the individual determinants of the remaining square minors.

$|K|$ = (1) x [1 x (1x0-0x1)] + (-1) x [1 x (2x0-0x0)] + (1) x [2 x (2x1-1x0)]

$|K|$ = (1) x [1 x (0)] + (-1) x [1 x (0)] + (1) x [2 x (2)]

$|K|$ = (1) x [0] + (-1) x [0] + (1) x [4] = 0 + 0 + 4 = 4

8) What is the inverse of matrix K?

$$K^{-1} = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}^{-1} = \frac{1}{|K|} \times \left[Adjoints \right]^T = \frac{1}{4} \times \begin{bmatrix} (1)x\begin{vmatrix} 1 & 0 \\ 1 & 0 \end{vmatrix} & (-1)x\begin{vmatrix} 2 & 0 \\ 0 & 0 \end{vmatrix} & (1)x\begin{vmatrix} 2 & 1 \\ 0 & 1 \end{vmatrix} \\ (-1)x\begin{vmatrix} 1 & 2 \\ 1 & 0 \end{vmatrix} & (1)x\begin{vmatrix} 1 & 2 \\ 0 & 0 \end{vmatrix} & (-1)x\begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} \\ (1)x\begin{vmatrix} 1 & 2 \\ 1 & 0 \end{vmatrix} & (-1)x\begin{vmatrix} 1 & 2 \\ 2 & 0 \end{vmatrix} & (1)x\begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} \end{bmatrix}^T$$

$$= \frac{1}{4} \times \begin{bmatrix} (1)x(1x0 - 0x1) & (-1)x(2x0 - 0x0) & (1)x(2x1 - 1x0) \\ (-1)x(1x0 - 2x1) & (1)x(1x0 - 2x0) & (-1)x(1x1 - 1x0) \\ (1)x(1x0 - 2x1) & (-1)x(1x0 - 2x2) & (1)x(1x1 - 1x2) \end{bmatrix}^T$$

$$K^{-1} = \frac{1}{4} \times \begin{bmatrix} (1)x(0) & (-1)x(0) & (1)x(2) \\ (-1)x(-2) & (1)x(0) & (-1)x(1) \\ (1)x(-2) & (-1)x(-4) & (1)x(-1) \end{bmatrix}^T = \frac{1}{4} \times \begin{bmatrix} 0 & 0 & 2 \\ 2 & 0 & -1 \\ -2 & 4 & -1 \end{bmatrix}^T$$

$$= \frac{1}{4} \times \begin{bmatrix} 0 & 2 & -2 \\ 0 & 0 & 4 \\ 2 & -1 & -1 \end{bmatrix} = \begin{bmatrix} 0 & .5 & -.5 \\ 0 & 0 & 1 \\ .5 & -.25 & -.25 \end{bmatrix}$$

9) What is the product of post multiplying matrix K by its inverse?

$$K \times K^{-1} = KK^T = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix} \times \begin{bmatrix} 0 & .5 & -.5 \\ 0 & 0 & 1 \\ .5 & -.25 & -.25 \end{bmatrix}$$

$$= \begin{bmatrix} (1x0 + 1x0 + 2x.5) & (1x.5 + 1x0 + 2x - .25) & (1x - .5 + 1x - 1 + 2x - .25) \\ (2x0 + 1x0 + 0x.5) & (2x.5 + 1x0 + 0x - .25) & (2x - .5 + 1x1 + 0x - .25) \\ (0x0 + 1x0 + 0x.5) & (0x.5 + 1x0 + 0x - .25) & (0x - .5 + 1x1 + 0x - .25) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The product of a matrix post (or pre) multiplied by its inverse is an identity matrix.

10) Write out the equations and solve the system of linear equations.

$$\begin{bmatrix} 3 \\ 3 \\ 1 \end{bmatrix} = K \times \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix} \times \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

Equations: $$\begin{Bmatrix} 3 = 1x_1 + 1x_2 + 2x_3 + 0 \\ 3 = 2x_1 + 1x_2 + 0x_3 + 1 \\ 1 = 0x_1 + 1x_2 + 0x_3 + 1 \end{Bmatrix}$$

The system of linear equations can be solved using matrix operations or algebraically solving the system of three equations, with three unknowns, for the unknowns.

Algebraically:

$$= \begin{Bmatrix} 3 = 1x_1 + 1x_2 + 2x_3 + 0 \\ 3 = 2x_1 + 1x_2 + 0x_3 + 1 \\ 1 = 0x_1 + 1x_2 + 0x_3 + 1 \end{Bmatrix} = \begin{Bmatrix} 3 = 1x_1 + 1x_2 + 2x_3 \\ 3 = 2x_1 + 1x_2 + 1 \\ 1 = 1x_2 + 1 \end{Bmatrix} = \begin{Bmatrix} 3 = 1x_1 + 1x_2 + 2x_3 \\ 3 - 1 - x_2 = 2x_1 \\ 1 - 1 = x_2 \end{Bmatrix}$$

$$= \begin{Bmatrix} 3 = 1x_1 + 1x_2 + 2x_3 \\ 1 - .5x_2 = x_1 \\ 0 = x_2 \end{Bmatrix} = \begin{Bmatrix} 3 = 1x_1 + 1x_2 + 2x_3 \\ 1 - .5 \times 0 = x_1 \\ 0 = x_2 \end{Bmatrix} = \begin{Bmatrix} 3 = 1x_1 + 1x_2 + 2x_3 \\ 1 = x_1 \\ 0 = x_2 \end{Bmatrix}$$

$$= \begin{Bmatrix} 3 = 1 \times 1 + 1 \times 0 + 2x_3 \\ 1 = x_1 \\ 0 = x_2 \end{Bmatrix} = \begin{Bmatrix} 3 - 1 = 2x_3 \\ 1 = x_1 \\ 0 = x_2 \end{Bmatrix} = \begin{Bmatrix} 1 = x_3 \\ 1 = x_1 \\ 0 = x_2 \end{Bmatrix} \Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

Matrix Operations:

$$\begin{bmatrix} 3 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix} \times \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 \\ 3 \\ 1 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix} \times \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad \Rightarrow \quad \begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix} \times \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\begin{bmatrix} 0 & .5 & -.5 \\ 0 & 0 & 1 \\ .5 & -.25 & -.25 \end{bmatrix} \times \begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix} \times \begin{bmatrix} 0 & .5 & -.5 \\ 0 & 0 & 1 \\ .5 & -.25 & -.25 \end{bmatrix} \times \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\begin{bmatrix} 0 \times 3 + .5 \times 2 - .5 \times 0 \\ 0 \times 3 + 0 \times 2 + 1 \times 0 \\ .5 \times 3 - .25 \times 2 - .25 \times 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad \Rightarrow \quad \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$