

# Structural Equation Modeling Lab 5

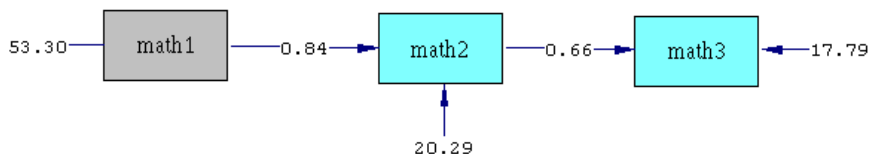
## In Class Modification Indices Example

### 1. Model specifications syntax

```
TI Modification Indices
DA NI=10 NO=0 MA=CM
RA FI='E:\Teaching\SEM S09\Lab 5\jsp162.psf'
SE
7 6 5 /
MO NX=1 NY=2 BE=FU GA=FI PS=SY
FR BE(1,2) GA(2,1)
PD
OU ND=4 SS EF MI RS
```

**Note.** LISREL specifies PS=SY when building syntax from the user drawn path diagram, but interprets it to be PS=DI, FR in the model specification for estimation (as can be seen in the section of parameter specifications in the output). It is recommended that PS=DI, FR is specified explicitly when constructing your own syntax for this model.

Figure 1. Temporal model path diagram with estimates



Chi-Square=35.47, df=1, P-value=0.00000, RMSEA=0.464

### 2. Calculating degrees of freedom

Degrees of freedom are determined by subtracting the number of free parameters we are estimating from the number of observed terms we are inputting. In this model the  $df = 1$ . This is because we are modeling one less path than what was modeled in the previous week. We are inputting 6 unique terms with our covariance matrix but we are estimating 5 parameters, leaving one observed term free to vary given the model specifications and parameter estimates.

$df = (\text{variances} + \text{covariances}) - (\text{parameters being estimated})$

$$df = \left( 3 \text{ var} + \frac{3 \times (3-1)}{2} \text{ cov} \right) - (\beta_{21}, \gamma_{12}, \text{var}(x_1), \text{var}(\zeta_1), \text{var}(\zeta_2))$$

$$df = (3 + 3) - (2 + 3) = 6 - 5 = 1$$

One degree of freedom will result in a model in which fit can now be assessed since the equations specifying the relational aspects are no longer just identified. Subsequently, LISREL will produce a series of goodness of fit statistics, which we shall investigate one at a time

throughout the course and will take a closer look at some later on in this example.

### 3. Fitted regression equations

For  $x_1 = \text{Math1}$ ,  $y_2 = \text{Math2}$  and  $y_1 = \text{Math3}$ :

$$\hat{y}_1 = 12.9864 + 0.6591y_2$$

$$\hat{y}_2 = 4.7819 + 0.8414x_1$$

### 4. Path diagram equations

$$y_1 = \alpha_1 + \beta_{12}y_2 + \zeta_1$$

$$y_2 = \alpha_2 + \gamma_{21}x_1 + \zeta_2$$

$$x_1 = x_1$$

These are the equations which are used to reproduce the originally observed variance-covariance matrix.

### 5. Reduced form

The reduced form for this model is similar to the previous reduced form with the notable difference that whereas before we had a multiple regression expressing math3 we now have a simple linear regression. Given,

$$y_1 = \alpha_1 + \beta_{12}y_2 + \zeta_1$$

$$y_2 = \alpha_2 + \gamma_{21}x_1 + \zeta_2$$

we can express  $y_1$  in terms of  $x_1$  by substituting  $y_2$ 's equation into the first one. This will produce the reduced form for  $y_1$ . Since we cannot

express  $x_1$  in any other terms, the reduced form for  $y_2$  remain the same as in the full model specification.

$$y_1 = \alpha_1 + \beta_{12} \times (\alpha_2 + \gamma_{21}x_1 + \zeta_2) + \zeta_1$$

$$y_1 = \alpha_1 + \beta_{12}\alpha_2 + \beta_{12}\gamma_{21}x_1 + \beta_{12}\zeta_2 + \zeta_1$$

$$y_1 = (\alpha_1 + \beta_{12}\alpha_2) + (\beta_{12}\gamma_{21}x_1) + (\beta_{12}\zeta_2 + \zeta_1)$$

$$y_1 = (\alpha_1 + \beta_{12}\alpha_2) + (\beta_{12}\gamma_{21})x_1 + (\beta_{12}\zeta_2 + \zeta_1)$$

## 6. T-test of coefficients

Every fitted parameter is presented in LISREL output in a three line form. The first line is the estimate of the parameter followed by its corresponding standard error (the second line in parentheses). The last line of this grouping is the  $t$ -value, calculated as the ratio of the estimate to the standard error. These T-values, similar to those seen in ordinary regression, are used to test the significances of the path coefficients. Let us consider  $\beta_{12}$ , the estimated path coefficient from the endogenous variable Math2 to the endogenous variable Math3.

BETA

	math3	math2
math3	--	0.6591 (0.0438) 15.0554
math2	--	--

**Note.** The matrix above shows the regression coefficient of an endogenous variable on another, as opposed to the gamma matrix which presents the regression coefficients of the endogenous variables in rows on the exogenous variables in columns.

Here  $\beta_{12} = .6591$  is the direct effect of Math2 on Math3. From the above matrix it follows that  $\beta_{12}$  has a standard error of .0438. The corresponding  $t$ -test (testing the hypothesis that the coefficient is equal to zero) can be thus computed as:

$$Z = \frac{\beta_{12} - 0}{s.e.(\beta_{12})} = \frac{.6591 - 0}{.0438} = 15.0554,$$

**Note.** Individual  $t$ -value can be seen in the path diagram by changing the estimate view to  $t$ -values.

Since in this approach we are interested in if the coefficient is significantly different from zero (i.e. no effect), zero is subtracted from the estimate in the numerator of the computational formula.

## 7. Standardized solutions

As a reminder, the standardized solutions for each estimate can be computed by multiplying the unstandardized coefficient by the ratio of the standard deviation of the predictor to that of the dependent variable. For instance if we wish to standardize  $\gamma_{21}$ , the path from Math1 to Math2, we would have to multiply the coefficient by the appropriate ratio of the standard deviations:

$$std(\gamma_{21}) = \gamma_{21} \times \frac{sd(Math1)}{sd(Math2)} = \gamma_{21} \times \frac{\sqrt{\text{var}(Math1)}}{\sqrt{\text{var}(Math2)}} = .8414 \times \frac{\sqrt{53.2988}}{\sqrt{58.0238}} = .8064$$

It is important to remember that whereas the unstandardized coefficient is scale dependent on both variables with which it is associated, the standardized coefficient no longer depends on the scales of both variables.

## 8. Goodness of fit

Goodness of fit statistics are measures used to assess the adequacy of the model with respect to its ability to reproduce the variance-covariance matrix. There are many different goodness of fit statistics in LISREL. LISREL will print these in the output as well as in a separate file labeled as the same as the model run with the extension .fit. If the model is just identified (saturated, d.f.=0) the goodness of fit statistics would be meaningless because the model will reproduce the original variance-covariance matrix perfectly. With no degrees of freedom there is nothing in the model left free to estimate and thus the assessment of the fit is unnecessary.

However, in our example, by having deleted one of the gamma paths in the saturated model, we are left with one degree of freedom and nontrivial goodness of fit assessments. For the time being, let us look at the following five statistics presented in the goodness of fit section of the output:

Degrees of Freedom = 1

Normal Theory Weighted Least Squares Chi-Square = 35.4710 (P = 0.0)

Minimum Fit Function Value ( $F_{\min}$ ) = 0.2489  
Minimum Fit Function Chi-Square = 40.0677  
Root Mean Square Residual (RMR) = 3.4893  
Standardized RMR = 0.07289

Perhaps the most commonly used goodness of fit statistic is the *Normal Theory Weighted Least Squares Chi-Square* (NTWLS  $\chi^2$ ). Note that LISREL includes this statistic at the bottom of the path diagram when the PD command is invoked. This chi-square statistic is a measure of the discrepancies between the observed variance-covariance matrix (for all of the observed variables included in the model) and the reproduced variance-covariance matrix. From earlier labs you have learned that the model equations are used to reproduce the variance-covariance matrix. The reproduced matrix is labeled *Covariance Matrix of Y and X* in the LISREL output. When fitted values and residuals are requested (by specifying RS in the OU command), LISREL will also produce a matrix labeled *Fitted Covariance Matrix*, which is the same as the *Covariance Matrix of Y and X*. The NTWLS  $\chi^2$  indicates the extent to which the observed matrix deviates from the model reproduced (implied) matrix. Contrary to our intuition (guided by our hypothesis testing habits) we desire the NTWLS  $\chi^2$  to be not significant. A non-significant NTWLS  $\chi^2$  means that the two matrices do not differ significantly from one another and that our specified model adequately reproduces the original variance-covariances.

The *Minimum Fit Function Chi-square* is computed as  $(N - 1) \times F_{\min}$ , where  $F_{\min}$  is the value of the fitting function (from either a Maximum Likelihood or a Generalized Least Squares procedure) at the convergence of the iterative estimation procedure. This value is largely used in the computation of the *Minimum Fit Function Chi-Square* (MFF  $\chi^2$ ), which tests the model for exact fit (i.e. the null hypothesis is that the model implied variance-covariance matrix is identical to the population matrix). The MFF  $\chi^2$  is computed as:

$$MFF \chi^2 = (N - 1) \times F_{\min}$$

$$MFF \chi^2 = (162 - 1) \times .2489 = 161 \times .2489 = 40.07$$

As with the NTWLS  $\chi^2$ , the MFF  $\chi^2$  is desired to be non-significant -- MFF  $\chi^2$  tests for exact fit.

The last two indices for goodness of fit considered in this example are the *Root Mean Squares Residuals* (RMR) and the Standardized RMR (SRMR) presented at the bottom of the goodness of fit section in the

output. The residuals for this model are the differences between the observed (sample) and fitted (reproduced) variance-covariance matrices. The RMR is a scale dependent indicator of how much the two matrices differ from one another:

$$RMR = \sqrt{\frac{\sum_{i \leq j}^p (\sigma_{ij} - \hat{\sigma}_{ij})^2}{p + \frac{p \times (p-1)}{2}}}$$

where  $\sigma_{ij}$ 's are the observed variances and non-redundant covariances,  $\hat{\sigma}_{ij}$ 's are the fitted variances and non-redundant covariances, and  $p$  is the number of observed variables in the specified model. Since the RMR is scale dependent it can be misleading when used for model evaluation. Therefore a standardized alternative can be considered, the SRMR. For a model with perfectly reproduced variances (as in the current example), the standardized form can be obtained by fitting the correlation matrix. This can be verified by using MA=KM, instead of MA=CM, in the DA command of the syntax. The resulting RMR with MA=KM is the same as the earlier SRMR. This value is not scale dependent and can now be generally evaluated. Kline (2005) as well as Raykov and Marcoulides (2006) suggest that SRMR less than .10 are favorable, which it is in our example.

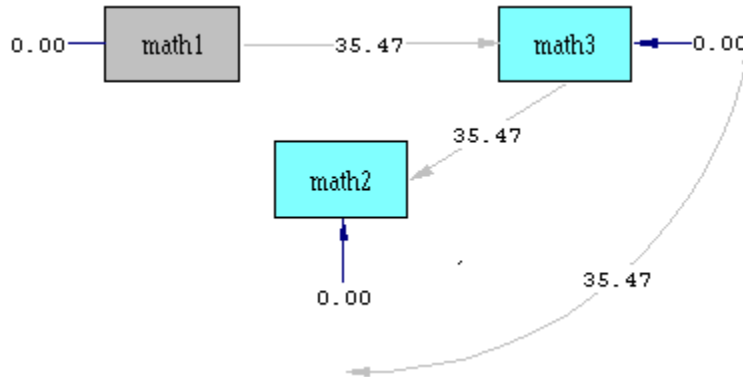
Sampling distributions of many goodness of fit statistics are usually not examined directly. Instead, rules of thumb are suggested (sometimes based on simulations).

## 9. Modification indices

Modification indices produced by LISREL indicate the expected change of Chi-square statistic when the specified model is modified by freeing one of the previously fixed parameters. The change is projected by considering a modification to the model by freeing one parameter at a time. As there is only one degree of freedom for the specified model, only a single parameter can be added, which renders the model saturated (just identified). Thus, freeing any of the fixed parameters produces the same modification index. If there are more degrees of freedom and we want to add or change more than one specification, the modification indices presented in the output should be interpreted as one change at a time. It is suggested that the parameter with the largest modification index may be freed first, but substantive interpretation of such modification should also be considered. Sequential modifications of the model can be made as in the forward stepwise regression procedure.

For every possible modification, three quantities are provided. The first reports the modification index for the path, which is the expected reduction of the NTWLS  $\chi^2$  by freeing (adding) the parameter. Figure 2 shows this relationship.

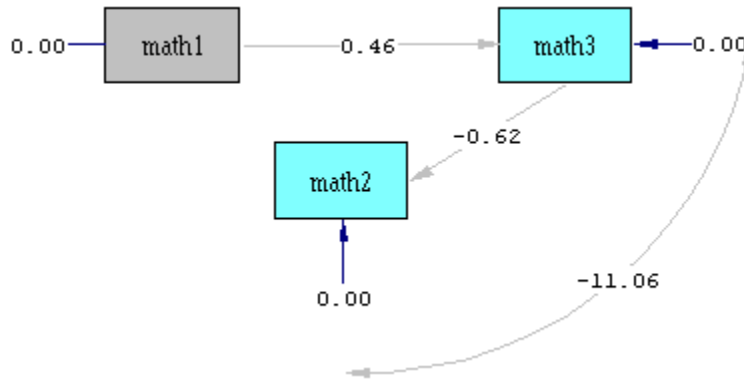
Figure 2. Modification indices



As can be seen in the figure, there are three possible modifications: adding a direct path from math1 to math3 (i.e., adding  $\gamma_{11}x_1$  to the equation for  $y_1$ , see the path diagram presented in section 4 earlier); adding a feedback path from math3 to math2 (i.e., adding  $\beta_{21}y_1$  to the equation for  $y_2$ , see the path diagram presented in section 4 earlier); and allowing the errors ( $\zeta_1$  and  $\zeta_2$ ) for the two equations to be correlated (i.e., freeing the covariance  $\phi_{12}$  in the  $\Psi$  matrix – by specifying PS=SY, FR in the MO command). Each of the possible modifications will result in a saturated model. This is consistent with the observation that the initial model has just one degree of freedom. As  $\chi^2$  for a saturated model (with zero degree of freedom) is zero, each modification will reduce  $\chi^2$  to zero (i.e., having a modification index equal to 35.47, which is the NTWLS  $\chi^2$  for the initial model).

The second matrix in every section of the modification indices provides the expected change of the unstandardized estimate of the parameter if modified alone. As each of the possible modifications is to change the parameter from zero, the expected change here is effectively the projected estimate of the parameter in the modified model. Figure 3 shows the projected estimates of the corresponding parameters. Here we can see that both the feedback path and the error covariance would be negative. Neither of which make much sense and constitute in fact a statistical modeling possibility rather than a sensible, theory supported, modification.

Figure 3. Expected changes



On the other hand, the gamma coefficient ( $\gamma_{11}$ ) is the path currently omitted from the saturated model fitted at the last lab (Lab 4, 2-11-09). Note this particular saturated model makes sense conceptually in the temporal context. As we can see from the expected change matrix, this modified model is expected to give the same estimate for  $\gamma_{11}$  as obtained for the Lab 4 model. It is important to see that the modified model will also change the estimate of  $\beta_{12}$  in the current model (its estimate will become same as that obtained at Lab 4). Note that LISREL also gives a summary of the modification indices by showing which of all of the possible (single) modifications is expected to result in the largest improvement in fit (as indicated by the reduction of  $\chi^2$ ). This is helpful when considering larger models with a greater number of degrees of freedom. However, it is not necessarily the best single modification (which is best supported conceptually) when equivalent modifications on different types of parameters (e.g., direct path, feedback path, or error covariances) are possible as seen this example.

## 10. Direct and indirect effects

In the present example, calculations of direct and indirect effects are fairly straightforward. In the temporal model done at lab 4, there are only two paths. This translates into two direct effects and one indirect effect.  $\beta_{12}$  is the direct effect of Math2 on Math3 and  $\gamma_{21}$  is the direct effect of Math1 on Math2. Since there is no path from Math1 to Math3, there is no direct effect of Math1 on Math3. However, there is an indirect effect of Math1 on Math3, which equals to the product  $\gamma_{21} \times \beta_{12}$  as shown in the last expression of the reduced form derived in section 5 earlier. Because the present model assumes no direct effect of Math1 on Math3, this is also the total effect of Math1 on Math3. Likewise, the direct effects of Math1 on Math2 and Math2 on Math3 are also equivalent to their corresponding total effects.