Structural Equation Modeling Lab 3

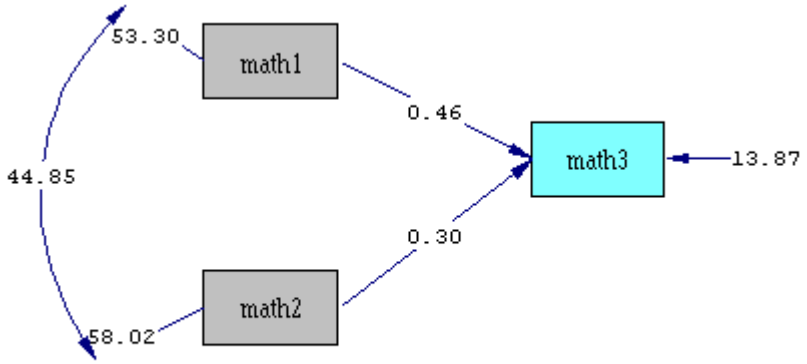
In Class Multiple Regression Example

1. First let us build the appropriate syntax.

```
TI Multiple Regression
Regressing Math3 on Math1 and Math2
DA NI=10 NO=0 MA=CM
RA FI='C:\ Your path here \jsp162_lab1_2-4-09.psf'
SE
 7 5 6 /
MO NX=2 NY=1 GA=FI PS=SY
FR GA(1,1) GA(1,2)
PD
OU
```

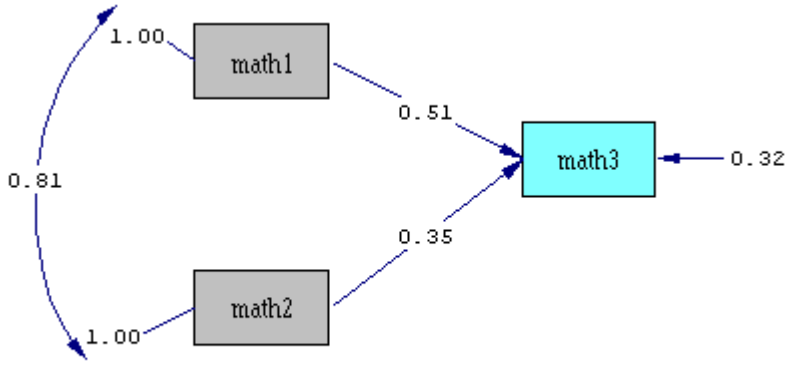
2. Next let us build run the syntax and look at the path diagram

   Unstandardized:



Chi-Square=0.00, df=0, P-value=1.00000, RMSEA=0.000

   Standardized:



Chi-Square=0.00, df=0, P-value=1.00000, RMSEA=0.000

3. Verify your syntax and output. Make sure that LISREL produced the correct estimates and that your model is what you wanted.

4. The resulting regression model is:

$$Math3 = \alpha_1 + \gamma_{11}Math1 + \gamma_{12}Math2 + \varepsilon_1$$

$$Math3 = 10.61 + .46 \times Math1 + .30 \times Math2$$

5. Let us look at the degrees of freedom:

df = (variances + no redundant covariances) – (#paths + #errors)

$$df = \left(3 + \frac{3 \times (3-1)}{2}\right) - \left(2\ paths + 1\ error + 1\text{cov} + 2\,\text{var}\right) = (3+3) - (6) = 6 - 6 = 0$$

6. The correlation between the two exogenous variables can be checked as follows:

$$cor(Math1,\ Math2) = \frac{\text{cov}(Math1,\ Math2)}{s_{Math1} \times s_{Math2}} = \frac{44.8473}{7.301 \times 7.617} = \frac{44.8473}{55.6117} = .8064$$

7. Since there are no degrees of freedom left the model is a perfect fit and the subsequent reproduced variance/covariance matrix should be perfectly replicated. Let us verify that that is the case.

$$\text{var}(Math1) = 53.2988$$

$$\text{var}(Math2) = 58.0238$$

$$\text{cov}(Math1,\ Math2) = 44.8473$$

$$\begin{aligned}
\text{cov}(Math1,\ Math3) &= Math1 \times (\alpha_1 + \gamma_{11}Math1 + \gamma_{12}Math2 + \zeta_1) \\
&= 0 + \gamma_{11} \times \text{var}(Math1) + \gamma_{11} \times \text{var}(Math1) + 0 \\
&= \gamma_{11} \times \text{var}(Math1) + \gamma_{11} \times \text{cov}(Math1,\ Math2) \\
&= .4586 \times 53.2988 + .3046 \times 44.8473 = 24.4428 + 13.6605 = 38.1033
\end{aligned}$$

$$\begin{aligned}
\text{cov}(Math2,\ Math3) &= Math2 \times (\alpha_1 + \gamma_{11}Math1 + \gamma_{12}Math2 + \zeta_1) \\
&= 0 + \gamma_{11} \times \text{cov}(Math2,Math1) + \gamma_{11} \times \text{var}(Math2) + 0 \\
&= \gamma_{11} \times \text{cov}(Math2,Math1) + \gamma_{11} \times \text{var}(Math2) \\
&= .4586 \times 44.8473 + .3046 \times 58.0238 = 20.5670 + 17.6740 = 38.2410
\end{aligned}$$

$$
\begin{aligned}
\text{var}(Math3) \quad &= (\alpha_1 + \gamma_{11}Math1 + \gamma_{12}Math2 + \zeta_1) \times (\alpha_1 + \gamma_{11}Math1 + \gamma_{12}Math2 + \zeta_1) \\
&= \alpha_1 \times (\alpha_1 + \gamma_{11}Math1 + \gamma_{12}Math2 + \zeta_1) \\
&+ \gamma_{11}Math1 \times (\alpha_1 + \gamma_{11}Math1 + \gamma_{12}Math2 + \zeta_1) \\
&+ \gamma_{12}Math2 \times (\alpha_1 + \gamma_{11}Math1 + \gamma_{12}Math2 + \zeta_1) \\
&+ \zeta_1 \times (\alpha_1 + \gamma_{11}Math1 + \gamma_{12}Math2 + \zeta_1) \\
&= 0 \\
&+ 0 + \gamma_{11}^2 \times \text{var}(Math1) + \gamma_{11}\gamma_{12} \times \text{cov}(Math1,\ Math2) + 0 \\
&+ 0 + \gamma_{11}\gamma_{12} \times \text{cov}(Math1,\ Math2) + \gamma_{12}^2 \times \text{var}(Math2) + 0 \\
&+ 0 + 0 + 0 + \text{var}(\zeta_1) \\
&= \gamma_{11}^2 \times \text{var}(Math1) + \gamma_{11}\gamma_{12} \times \text{cov}(Math1,\ Math2) \\
&+ \gamma_{11}\gamma_{12} \times \text{cov}(Math1,\ Math2) + \gamma_{12}^2 \times \text{var}(Math2) \\
&+ \psi_1 \\
&= \gamma_{11}^2 \times \text{var}(Math1) + 2 \times \gamma_{11}\gamma_{12} \times \text{cov}(Math1,\ Math2) \\
&+ \gamma_{12}^2 \times \text{var}(Math2) + \psi_1 \\
&= .4586^2 \times 53.2988 + 2 \times .4586 \times .3046 \times 44.8473 \\
&+ .3046^2 \times 58.0238 + 13.8722 \\
&= .2103 \times 53.2988 + .2794 \times 44.8473 + .0928 \times 58.0238 + 13.8722 \\
&= 11.2087 + 12.5303 + 5.3846 + 13.8722 = 42.9958
\end{aligned}
$$